4

Properties of addition and subtraction of integers

Closure under addition

For any two integers a and b, a + b is an integer. The integers are closed under addition.

Example:

(1) 14 + 25 = 39, which is an integer

(2) -83 + 49 = -34, which is an integer

5

Closure under subtraction

If a and b are two integers then a - b is also an integer i.e., Integers are closed under subtraction.

Example

(1) 8 - 15 = -7, which is an integer.

(2) -20 - 23 = -43, which is an integer.

6

Commutative property

For any two integers a and b we can say

a + b = b + a

i.e., Addition is commutative for integers.

Example

For any two integers, -7 and 5

-7 + 5 = -2, 5 + (-7) = -2

∴ -7 + 5 = 5 + (-7)

7

Subtraction is not commutative for integers

Example:

For any two integers, -11 and 8

-11- 8= -19 and -8-(-11) = -8 + 11 = 3

-11 - 8 $\neq$8 - (-11)

So, for any two integers a and b, a - b $\neq$ b - a

8

Associative property

For any integers a, b and c, we can say

a + (b + c) = (a + b) + c

i.e., Addition is associative for integers.

Example:

For any three integers -4, -3, -5

-4 + (-3) + (-5) = -4 + (-8) = -4 - 8 = -12

(-4+(-3)) + (-5) = (-7) + (-5) = -7 - 5 = -12

$\therefore$ -4 + (-(3) + (-5)) = (-4 + (-3)) + (-5)

9

Additive Identity

For any integer a, we have

a + 0 = a = 0 + a

i.e., integer 0 (zero) is the additive identity or identity under addition for integers.

Example:

For an integer -10 we have

-10 + 0 = -10 ,0 + (-10) = -10

$\therefore$ -10 + 0 = -10 = 0 + (-10)

10

Multiplication of integers

Multiplication of a positive integer and a negative integer

While multiplying a positive integer and a negative integer, we multiply them as whole numbers and part a minus sign (-) before the product. We thus get a negative integer.

Therefore, the product of a positive and negative integer is a negative integer.

For any two positive integers a and b we can say

a × (-b) = (-a) × b = -(a×b)

Example:

$(-4) \times (8) = 4 \times (-8) = -(4 \times 8)$= -32, a negative integer

11

Multiplication of two negative integers

The product of two negative integers is a positive integer. We multiply the two negative integers as whole numbers and put the positive sign before the product.

For any two positive integers a and b,

(-a) × (-b) = a × b

Example:

(-13) × (-15) = 195

12

Product of three or more negative integers

⋆ If the number of negative integers in a product is even then the product is a positive integer.

⋆ If the numbers of negative integers in a product is odd, then the product is a negative integer.

Example:

⋆ (-5) (-4) (-6) (-7) = 840 (Positive integer since no. of negative integers is even)

⋆ (5) (-4) (-6) (-7) = -840 (Negative integer since the no. of negative integers is odd)

Also,

$(-1) \times (-1) \times (-1) \times \ldots$ multiplied even no. of times = +1

$(-1) \times (-1) \times (-1) \times \ldots$ multiplied odd no. of times = -1

13

Properties of multiplication of integers

Closure under multiplication

Integers are closed under multiplication i.e., in general,

a $\times$ b is an integer, for all integers a and b.

Example:

$15 \times (-4) = -60$, which is an integer.

$(-14) \times (-19) = 266$, which is an integer.

14

Commutativity of multiplication

Multiplication is commutative for integers.

i.e., in general, for any two integers a and b,

a $\times$ b = b $\times$ a

Example:

$(5) \times (-4) = -20, (-4) \times (5) = -20$

$\therefore (5) \times (-4) = (-4) \times (5)$

15

Multiplication by zero

For any integer a, a$\times$ 0 = 0 $\times$ a = 0

i.e., The product of an integer (positive or negative) and zero is zero.

Example:

$6 \times 0 = 0 \times 6 = 0$

$-15 \times 0 = 0 \times (-15) = 0$

16

Multiplicative identity

For any integer a, a $\times$ 1 = 1 $\times$ a = a

i.e., 1 is the multiplicative identify for integers

Example:

$-4 \times 1 = 1 \times (-4) = -4$

$7 \times 1 = 1 \times 7 = 7$

⋆ 0 is the additive identify where as 1 is the multiplicative identify for integers.

⋆ We get additive inverse of an integer a when we multiply (-1) to a, i.e., a $\times(-1) = (-1)\times$ a = -a

17

Associativity for multiplication

Multiplication is associative for integers

i.e., for any three integers a,b and c

(a $\times$ b) $\times$ c = a $\times$ (b $\times$ c)

Example:

$(5 \times (-4)) \times 3 = (-20) \times 3 = -60$

$5 \times ((-4) \times 3) = 5 \times (-12) = -60$

$(5 \times (-4)) \times 3 = 5 \times ((-4) \times 3) = -60$

⋆ The product of three integers does not depend upon the grouping of integers and this is called the associative property for multiplication of integers.

18

Distributive property

⋆ Distributivity of multiplication over addition is true for integers.

i.e., For any integers a, b and c.

a $\times$ (b + c) = a $\times$ b + a $\times$ c

Example:

$$10 \times [6 + (-2)] = 10 \times (4) = 40$$
$$10 \times 6 + 10 \times (-2) = 60 - 20 = 40$$
$$\therefore 10 \times [6 + (-2)] = 10 \times 6 + 10 \times (-2)$$

⋆ Also,for any three integers a, b and c

a $\times$ (b - c) = (a $\times$ b) - (a $\times$ c)

Example:

consider -5, -4, 6

5 $\times$ (-4 - 6) = 5(-10) = -50

5(-4) - 5(6) = -20 - 30 = -50

$\therefore$ 5 $\times$ (-4 - 6) = 5 $\times$ (-4) - 5 $\times$ (6)

⋆ The properties of commutativity, associativity under addition and multiplication, and the distributive property help us to make our calculations easier.

19

Division of integers

Division is the inverse operation of multiplication.

⋆ When we divide a negative integer by a positive integer, we divide them as whole number and then put a minus sign(-) before the quotient. We thus, get a negative integer.

Example:

(-45) $\div$ 9 = -5

⋆ When we divide a positive integer by a negative integer, we first divide them as whole numbers and then put a minus sign (-) before the quotient. The result will be a negative integer.

Example:

$$50 \div (-5) = -10$$

⋆ In general, for any two positive integers a and b

a $\div$ (-b) = (-a) $\div$ b where b $\neq$ 0

⋆ When we divide a negative integer by a negative integer, we first divide them as whole numbers and then put a positive sign (+). The result will be a positive integer.

Example:

(-32) ÷ (-4) = 8

⋆ In general, for any two positive integers a and b

(-a) ÷ (-b) = a÷ b where b $\neq$ 0

20

Properties of division of integers

⋆ Division is not commutative for integers.

Example:

$(-8) \div (-4) = 2,$an interger

$(-4) \div (-8) = \frac{-4}{-8}$, not an integer

$(-8) \div (-4) \neq (-4) \div (-8)$

⋆ For any integer a, a ÷ 0 is not defined but 0 ÷ a = 0 for a $\neq$ 0

⋆ Any integer divided by 1 gives the same integer. In general, for any integer a,

a ÷ 1 = a

Example:

$5 \div 1 = 5$

⋆ If any integer is divided by (-1) it does not give the same integer.

Example:

$(-8) \div (-1) = 8$

$8 \neq -8$

⋆ Division is not associative for integers.

Example:

Consider -32, 8, -2

$(-32 \div 8) \div (-2) = (-4) \div (-2) = 2$

$-32 \div (8 \div (-2)) = (-32) \div (-4) = 8$

$\therefore (-32 \div 8) \div (-2) \neq (-32) \div (8 \div (-2))$

For CBSE, ICSE and all State Boards